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The high temperature phase transition in SU(N) gauge theories

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Abstract: We calculate the continuum value of the deconfining temperature in units of the string tension for SU(4), SU(6) and SU(8) gauge theories, and we recalculate its value for SU(2) and SU(3). We find that the N-dependence for $2 \leq N \leq 8$ is well fitted by $T_c/\sqrt{\sigma} = 0.596(4) + 0.453(30)/N^2$, showing a rapid convergence to the large-N limit. We confirm our earlier result that the phase transition is first order for $N \geq 3$ and that it becomes stronger with increasing N. We also confirm that as N increases the finite volume corrections become rapidly smaller and the phase transition becomes visible on ever smaller volumes. We interpret the latter as being due to the fact that the tension of the domain wall that separates the confining and deconfining phases increases rapidly with N. We speculate on the connection to Eguchi-Kawai reduction and to the idea of a Master Field.

Keywords: 1/N Expansion, Confinement, Lattice Gauge Field Theories.

Dedicated to the memory of Ian Kogan, friend and colleague.

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Contents

1. Introduction

In a recent letter [1] we presented lattice calculations of the finite temperature deconfining transition in SU(4) and SU(6) gauge theories. These calculations showed that the phase transition is first order and becomes more strongly so as N grows. Our continuum extrapolations, taken together with older SU(2) and SU(3) results, showed that the $O(1/N^2)$ corrections to the $N = \infty$ limit appear to be small, even for SU(2). Finally, our finite volume studies showed that the finite volume corrections decrease as N grows and we speculated on the simple picture that this suggested for SU($N = \infty$).

In this paper we will provide more accurate and more extensive calculations that confirm and make more precise our earlier study. For SU(4) and SU(6) we extend the range of lattice spacings at which we perform the calculations, thus allowing more reliable and precise continuum extrapolations. In addition we perform some calculations in SU(8) to strengthen our control of the $N \to \infty$ limit. We also perform some SU(2) and SU(3) calculations, both to check that our methods produce results in agreement with older (but still accurate) calculations and to investigate the crucial volume dependence in more detail. In addition we perform dedicated string tension calculations for all the above SU(N) groups, generally at a value of the lattice coupling very close to the phase transition. This again improves the accuracy of our calculation and, as a by-product, leads to significant changes to the older continuum extrapolations of the SU(2) and SU(3) deconfining temperatures.

The plan of the paper is as follows. In the next section we briefly summarise the details of the lattice setup. We then describe how we will identify the location and nature of the phase transition. We follow this with the details of our lattice results. We focus on $T_c/\sqrt{\sigma}$, the value of the deconfining temperature in units of the string tension, on the order of the transition, on the (euclidean) latent heat, where first order, and on the N-dependence of the finite volume corrections. We finish with some conjectures about the nature of physics at $N = \infty$, and with some conclusions.

We have, in addition, studied how topological fluctuations and (k-)string masses vary across the phase transition, how the bulk transition and the thermodynamic latent heat vary with N, and other aspects of the transition. These, together with more details about the technical aspects of our calculations, will be presented in a later publication [2]. For some comments on the interest of SU($N \to \infty$) gauge theories [3], some references to relevant lattice calculations, and some speculations [4] on the nature of the deconfining transition, we refer the reader to our earlier paper [1].

2. Lattice setup

Our lattice calculations are entirely conventional. We use hypercubic, periodic lattices with SU(N) matrices, U_l, assigned to the links l. We use the usual plaquette action

$$S = \beta \sum_p \left\{ 1 - \frac{1}{N} \operatorname{Re}\operatorname{Tr} U_p \right\}, \tag{2.1}$$

where U_p is the ordered product of the SU(N) matrices around the boundary of the plaquette p, and which appears in the euclidean path integral as e^{-S}. This lattice action becomes the usual Yang-Mills action with

$$\beta = \frac{2N}{g^2} \tag{2.2}$$

in the continuum limit. The coupling g^2 is the bare coupling and, on a lattice of spacing a, may be regarded as a running coupling, $g^2(a)$, defined at the length scale a at which the action is defined. So to decrease a we decrease $g^2(a)$ and hence increase β (once a is small enough). As we vary N we expect [3], that we will need to keep constant the 't Hooft coupling, λ, and its inverse, γ,

$$\lambda(a) \equiv g^2(a) N \, ; \qquad \gamma \equiv \frac{1}{\lambda} = \frac{\beta}{2N^2} \tag{2.3}$$

for a smooth large-N limit. Non-perturbative calculations [5, 6] support this expectation.

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A periodic lattice whose size is $L^3 L_t$ with $L \gg L_t$ may be used to calculate thermal averages of the field theory at a temperature

$$a(\beta)T = \frac{1}{L_t}\,. \tag{2.4}$$

We can therefore study the deconfining transition by fixing L_t and varying β so that T passes through T_c. For simplicity we shall refer to the 3+1 dimensional system as being at the temperature T given by eq. (2.4), although this can be deceptive (see below) and one needs to remember that the $D = 3 + 1$ system is no more than a theoretical framework for calculating thermal averages in a given three dimensional spatial slice at this value of T.

The simulations are performed with a combination of heat bath and over-relaxation updates, as described in [5, 6]. We update all the $N(N-1)/2$ SU(2) subgroups of the SU(N) link matrices.

3. The phase transition

We begin with some general comments about the deconfining phase transition and then we discuss how this phase transition appears in a finite volume and how one can extrapolate to the desired $V = \infty$ thermodynamic limit. The discussion here will be heuristic, but can be formalised through standard finite size scaling theory, both for first [7] and for second [8] order transitions. (We will summarise relevant aspects of this approach in our forthcoming paper [2].)

3.1 General remarks

The deconfining phase transition turns out to be second order for SU(2) and first order for SU($N \geq 3$). For a second order transition we have a diverging correlation length as $T \to T_c$ while for a first order transition various quantities will suffer a discontinuity at $T = T_c$ (including the correlation length which in general will be finite). This clear-cut distinction only holds in the $V = \infty$ thermodynamic limit. On a finite volume, as in our numerical calculations, the two types of transition can be confounded and so a careful finite volume study is obligatory. We shall describe later on in this section how we analyse the two kinds of phase transition.

We are of course interested in the continuum limit of this phase transition. The continuum limit of the lattice theory is described by a 2'nd order phase transition at $\beta = \infty$. This means that when we study the first order deconfining transition on the lattice, it will always appear to be weak; for example the latent heat is $O(a^4)$ and therefore small. Even on the coarser lattice spacings that are close to the bulk transition, which is first order for larger N, its latent heat is typically 1/50'th of the latter's. This is because the deconfining phase transition is a physical transition that only affects fluctuations on physical length scales, and leaves untouched the short distance fluctuations that usually dominate the values of composite operators. A typical lattice transition, like the bulk transition, affects fluctuations on all length scales. So to judge whether the first order phase transition is strong or weak we should compare the latent heat to a physical quantity, such as the $T = 0$ gluon condensate.

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The phase transition is characterised by the fact that for $T < T_c$ the confining string tension is non-zero, $\sigma \neq 0$, while for $T > T_c$ we have $\sigma = 0$. To calculate the string tension we calculate the free energy of two static sources in the fundamental representation placed a distance r apart, $F_{s\bar{s}}(r)$. Inserting such minimally coupled static sources into the action is equivalent to calculating the correlator of two Polyakov loops $\langle l_p^\dagger(r) l_p(0)\rangle$ where

$$l_p(\vec{n}_s) = \prod_{n_t=1}^{n_t=L_t} U_0(\vec{n}_s, n_t)\,. \tag{3.1}$$

(Here we introduce the notation, $U_\mu(n)$, for the matrix on the link emanating from site n in direction μ.) In the confining phase at temperature $T = 1/aL_t$

$$e^{-\frac{F_{s\bar{s}}(r)}{T}} = \langle l_p^\dagger(r = an_z) l_p(0)\rangle \stackrel{r\to\infty}{\propto} e^{-a^2\sigma(T) n_z L_t} \qquad T < T_c \tag{3.2}$$

where, to be definite, we take correlations in the z-direction. Since the operators l_p are localised in a given z-slice, this correlator can be expressed in terms of the eigenstates and eigenvalues of the transfer matrix T_z that translates us in the z-direction:

$$\langle l_p^\dagger(r = an_z) l_p(0)\rangle = \sum_n |\langle vac|l_p^\dagger|n\rangle|^2 e^{-am_n n_z} \tag{3.3}$$

where $T_z|n\rangle = e^{-am_n}|n\rangle$. If the lightest state in the sum is m_l then

$$\langle l_p^\dagger(r = an_z) l_p(0)\rangle \stackrel{n_z\to\infty}{\propto} e^{-am_l n_z}\,. \tag{3.4}$$

We see from eqs. (3.2), (3.4) that

$$am_l = a^2\sigma(T) L_t \tag{3.5}$$

and the fact that it is $\propto L_t$ indicates that this state of the z-transfer matrix is the lightest flux loop that winds once around the t-torus. Once we have determined that the interesting quantity is this lightest flux loop, we can alter the operator in the correlator to make the calculation more efficient. For example, we smear/block [5, 6] the links to have a better overlap onto the extended lightest state and we take $p_x = p_y = 0$ sums of these smeared Polyakov loops so as to exclude explicitly the more energetic states with $\vec{p} \neq 0$.

In the confining phase the Polyakov loop cannot have an overlap onto a non-winding state since otherwise we would lose the factor of L_t in the exponent of eq. (3.2). This constraint is enforced by a $Z_N \subset \mathrm{SU}(N)$ symmetry of the action and integration measure:

$$U_0(n_s, n_t) \to zU_0(n_s, n_t) \qquad \forall n_s \tag{3.6}$$

where $z \in Z_N$ is any one of the N'th roots of unity and n_t is some fixed value that one can freely choose. This symmetry ensures that $\langle l_p\rangle = 0$ since $l_p \to zl_p$ under the symmetry transformation.

In the deconfined phase $\sigma(T) = 0$ and we therefore lose the factor of n_z in the exponent of eq. (3.2) and so, in general, $\lim_{r\to\infty}\langle l_p^\dagger(r) l_p(0)\rangle \neq 0$, i.e. $\langle l_p\rangle \neq 0$. In terms of the Z_N symmetry described above, this is only possible if that symmetry is spontaneously broken.

This tells us that for $T > T_c$ there are N degenerate deconfined vacua characterised by $\langle l_p \rangle = ce^{i2k\pi/N}$, $k = 0, \ldots, N-1$ for c some real positive number. At very high T, where by asymptotic freedom $g^2(T) \ll 1$, one can calculate the tensions of the domain walls separating these vacua in perturbation theory [9]. It is important to note that what we actually know is that this Z_N symmetry breaking occurs in the $D = 3+1$ lattice system as one passes from $l_t \equiv aL_t > 1/T_c$ to $l_t < 1/T_c$, and indeed in the continuum limit of that system. It is not however at all clear that it is actually a feature of the field theory at high T. That is to say, no-one has (we believe) succeeded in showing that this symmetry breaking is encoded in any thermal averages in a way that displays the existence of such Z_N bubbles and domain walls in the hot field theory [10]. To avoid tedious circumlocution, we shall usually speak of our $D = 3+1$ system as being at a temperature $T = 1/aL_t$ although it is actually a framework within which to calculate thermal averages in a given spatial slice, and so one must always be careful when ascribing the detailed dynamics of this $3+1$ dimensional system to the hot field theory.

In summary, the natural order parameter for the deconfining phase transition is the average Polyakov loop, $\langle l_p \rangle$, and the mass of the flux loop that winds around the t-torus is the natural mass to focus upon, since it is simply proportional to $\sigma(T)$.

3.2 Characterising first order transitions

If the deconfining transition is first order, the finite-T effective potential will have both a confining vacuum and N degenerate deconfined vacua. At $T = T_c$ all of these vacua are degenerate. On a finite volume there will be a finite range of T around T_c in which the system has a significant probability to be in both confining and deconfined vacua, occasionally tunnelling between them. As we move away from $T = T_c$, or $\beta = \beta_c$, the degeneracy lifts and the confining/deconfined vacuum energies will split by some ΔE. The relative probabilities of the two vacua are given by $e^{-\beta \Delta E}$. Since $\Delta E \propto V$ we see that the finite volume transition region is typically $\delta\beta \sim O(1/V)$. To calculate the $V = \infty$ value of β_c, and hence of T_c, we must first make a choice of β_c on a finite volume, so that we can extrapolate it. A sensible criterion will use a value of $\beta_c(V)$ that is in the transition region, and therefore we generically can expect $\beta_c(V) = \beta_c(\infty) + O(1/V)$. We shall see examples of this below.

In an infinite volume the average plaquette will jump across the transition and the location of this jump defines for us the (pseudo-)critical coupling $\beta_c(V = \infty)$ and the critical temperature $a(\beta_c)T_c = 1/L_t$. In a finite volume the change will be continuous but rapid (if V is not too small). A useful quantity to characterise this behaviour is the specific heat $C(\beta)$:

$$\frac{1}{\beta^2} C(\beta) \equiv \frac{\partial}{\partial \beta} \langle \bar{u}_p \rangle = N_p \langle \bar{u}_p^2 \rangle - N_p \langle \bar{u}_p \rangle^2 \tag{3.7}$$

where $u_p \equiv \mathrm{Re}\,\mathrm{Tr}\, U_p/N$ and $\bar{u}_p$ is the average value of u_p for a given lattice field, and $N_p = 6L^3 L_t$ is the total number of plaquettes. The maximum of the specific heat is the maximum of $\partial \langle \bar{u}_p \rangle / \partial \beta$ (neglecting the weakly varying β^2 factor) and so the β at which it occurs provides at least one sensible definition of $\beta_c(V)$.

Let $\bar{u}_{p,c}$ and $\bar{u}_{p,d}$ be the average plaquette value for a lattice field in the confined and deconfined phase respectively, at $\beta = \beta_c(V)$. This value will fluctuate around its average value, $\langle \bar{u}_{p,c} \rangle$ or $\langle \bar{u}_{p,d} \rangle$, by an amount that is $O(1/\sqrt{N_p})$. As $V \to \infty$ we can neglect these fluctuations and we can also neglect the very rare fields where the system is tunnelling from one phase to another. In that case it is easy to see that the maximum of $C(\beta)$ occurs when the lattice fields are equally likely to be in the confined and deconfined phases, and in that case

$$\lim_{V \to \infty} \frac{1}{\beta_c^2 N_p} C(\beta_c) = \frac{1}{4} (\langle \bar{u}_{p,c} \rangle - \langle \bar{u}_{p,d} \rangle)^2 = \frac{1}{4} L_h^2 \tag{3.8}$$

where L_h is the latent heat per site of the phase transition and β_c denotes the value of β at which the maximum of $C(\beta)$ occurs. (Note that this latent heat is that of the transition of the $D = 3 + 1$ system; it is related to, but differs from, the latent heat of the finite-T transition of the field theory.) The V-dependence in eq. (3.8)

$$\lim_{V \to \infty} C(\beta_c) \propto V \tag{3.9}$$

is a direct consequence of the discontinuity in $\bar{u}_p$ and so is characteristic of a first order transition.

At a finite but large value of V, $\bar{u}_p$ will fluctuate around $\langle \bar{u}_p \rangle$ by an amount $O(1/\sqrt{N_p}) = O(1/\sqrt{V})$, i.e. $\langle \bar{u}_p^2 \rangle - \langle \bar{u}_p \rangle^2 = O(1/V)$. (We can neglect the exponentially suppressed tunnelling fields.) It is then easy to show that $C(\beta_c^C)$ will receive a correction that is $O(1/V)$, as long as the fluctuations in the confined and deconfined phases differ. Thus we should expect the leading large-volume correction to both $C(\beta_c^C)$ and $\beta_c^C(V)$ to be $O(1/V)$.

As we pointed out in section 3.1, the natural order parameter for the deconfining transition is the average Polyakov loop, $\langle l_p \rangle$. On a finite volume however, $\langle l_p \rangle = 0 \ \forall T$ because of the tunnelling between the N deconfined vacua in the deconfined phase. (This is simply the statement that one only has spontaneous symmetry breaking in an infinite volume.) To finesse this problem we follow usual practice and replace $\langle l_p \rangle$ by $\langle |\bar{l}_p| \rangle$ where $\bar{l}_p$ is the average of the Polyakov loop over a single lattice field. Clearly $\langle |\bar{l}_p| \rangle \neq 0 \ \forall T > T_c$. The price one pays is that now $\langle |\bar{l}_p| \rangle \neq 0$ also for $T < T_c$. However it is $O(1/\sqrt{V})$ in the confining phase and for a large volume this is a small price to pay. We can use this to define a quantity analogous to $C(\beta)$, where we replace the plaquette with the Polyakov loop. This is the normalised Polyakov loop susceptibility χ_l

$$\frac{\chi_l}{V} = \langle |\bar{l}_p|^2 \rangle - \langle |\bar{l}_p| \rangle^2 . \tag{3.10}$$

Just as with the specific heat we can define $\beta_c(V)$ to be the value of β at which χ_l has its maximum. At finite V the two definitions of $\beta_c(V)$ will differ by $O(1/V)$ terms, but will coincide in the thermodynamic limit. (We shall refer to them as β_c^χ and β_c^C when we wish to distinguish them.) Again we have the characteristic first order behaviour $\chi_l \propto V$ and corrections down by one power of V:

$$\lim_{V \to \infty} \frac{\chi_l}{V} = c_0 + \frac{c_1}{V} + \cdots \tag{3.11}$$

Just as for the specific heat, the maximum of χ_l occurs where the system is equally likely to be in the confined and deconfined phases (for $V \to \infty$). One might think that a more

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natural criterion would be one where the deconfined phase was N times more probable than the confined one (simply because of the relative multiplicity of the former) and that it would therefore have smaller $O(1/V)$ corrections. However we have not been able to construct a useful measure with such a behaviour to test this possibility.

As we remarked above, we expect the $V = \infty$ discontinuity to be smeared over an $O(1/V)$ transition region in a finite volume, so that definitions of $\beta_c(V)$ that use, for example, the peak of $C(\beta)$ or χ_l will have corrections that are $O(1/V)$. In physical units, we expect the correction in the continuum limit to be

$$\frac{T_c(\infty) - T_c(V)}{T_c(\infty)} \overset{V\to\infty}{=} \frac{h'}{VT_c^3} + \cdots \tag{3.12}$$

Inserting $T_c(V) = 1/a(\beta_c(V))L_t$ and Taylor expanding $a(\beta_c(V))$ around $V = \infty$ this translates, on an L^3L_t lattice, to the conventional finite size extrapolation [11]

$$\beta_c(\infty) - \beta_c(V) \overset{V\to\infty}{=} h\frac{L_t^3}{L^3} \tag{3.13}$$

where

$$h = \frac{-h'}{\frac{d}{d\beta}\ln a(\beta = \beta_c)} = \frac{-2N^2h'}{\frac{d}{d\gamma}\ln a(\gamma_c)}\,. \tag{3.14}$$

Thus if the leading finite-V correction is to be finite as $V \to \infty$ we should find $h \propto N^2$. In [1] we found some evidence that h is constant implying that the physical finite size correction vanishes as $h' \propto 1/N^2$ when $N \to \infty$. Note that since h' is a dimensionless physical quantity, it receives lattice corrections that are $O(a^2)$. The value of h on the other hand also depends on how a varies with β, i.e. on the violation of asymptotic scaling.

3.3 Characterising second order transitions

If the deconfining transition is second order, the string tension $\sigma(T)$ will vanish smoothly as $T \to T_c^-$. The finite-T effective potential will have a single minimum for $T < T_c$ with a quadratic term that vanishes at $T = T_c$. Thus at this temperature there is a diverging correlation length. This will be the inverse mass of the flux loop that winds around the t-torus; the vanishing at T_c of this mass follows from eq. (3.5) and from the vanishing of the string tension. For $T > T_c$ the former minimum at the origin becomes a maximum and a new minimum smoothly develops at a non-zero value of the order parameter $\langle l_p \rangle$. For SU(N) there would be N degenerate such minima (as discussed in section 3.1); however we know that only SU(2) is second order so we can specialise to that case, in which case we have two deconfined vacua characterised by $\langle l_p \rangle \propto \pm 1$.

Because the effective potential becomes flat near the transition, there will be large fluctuations. These will however be smooth, in contrast to the large tunnelling fluctuations of a first order transition. As T increases above T_c one will begin to have distinct tunnelling transitions, but these will be between the deconfined vacua.

Just as for first order transitions, $C(\beta)$ or χ_l diverge at T_c but the V dependence of this divergence is different and can be used to characterise the transition. Consider for

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example the specific heat. We may write it as

$$\frac{1}{\beta^2}C(\beta) = \frac{\partial}{\partial\beta}\langle\bar{u}_p\rangle = N_p\langle\bar{u}_p^2\rangle - N_p\langle\bar{u}_p\rangle^2 = \sum_p\langle(u_p - \langle u_p\rangle)(u_{p_0} - \langle u_{p_0}\rangle)\rangle\,, \tag{3.15}$$

where u_{p_0} denotes the value of the plaquette at fixed position p_0 on the lattice. Since the correlation length diverges as $T \to T_c$, we expect that the correlation function of the plaquettes will vary as $\stackrel{r\to\infty}{\propto} e^{-m_0(T)r}$ where $m_0(T) \to 0$ as $T \to T_c$ and $V \to \infty$. Thus

$$C(\beta_c(V)) \stackrel{V\to\infty}{\propto} V^{\zeta} \tag{3.16}$$

where $\zeta \le 1$ depends on the critical exponents of the transition through $\zeta = \gamma/\nu d$, using the standard notation. A similar derivation can be carried out for the loop susceptibility, χ_l. All this, as well as the V-dependence of $\beta_c(V)$, can be systematically analysed within the context of finite size scaling theory [8].

3.4 Tunnelling between phases

On an L^3L_t lattice with $L_t \ll L$ we can view as follows the tunnelling that occurs between phases that have nearly the same free energy. An intermediate step required for tunnelling is that the L^3 periodic volume be split into two roughly equal domains each in one of the two phases. Separating these will be two domain walls. Since these walls have a finite tension the most probable configuration will be the one that minimises the area, so they will extend right across the short t-torus and across two of the spatial tori. The probability $P_W(T)$ of forming such a bubble with two spatial walls of size $l^2 = (aL)^2$ is basically

$$P_W(T) \propto e^{-\frac{\text{energy walls}}{T}} \propto e^{-\frac{2\sigma_W l^2}{T}} \propto e^{-2a^3\sigma_W L^2 L_t} \tag{3.17}$$

where σ_W is the tension of the wall (i.e. energy per unit area).

We see from the L-dependence in eq. (3.17) that the probability of fields with such large domain walls is exponentially suppressed as the volume grows. This means that the 'time' the system spends in the transition between two phases is very small compared to the time it remains in one or the other phase: the transitions are rapid. It also means that the transitions are increasingly rare as L grows. These rare, large tunnelling fluctuations clearly distinguish between first and second order transitions once we are on a large enough volume. (Such tunnelling is also of course a feature of spontaneous symmetry breaking in a finite volume.)

This provides a qualitative description of the transition between confined and deconfined phases at $T = T_c$ and between the N degenerate deconfined vacua for $T > T_c$. In the latter case, for $T \gg T_c$ where $g^2(T) \ll 1$, one can use perturbation theory to evaluate the wall tension. One finds [9] for the wall tension between the $k = 0$ and k'th deconfined phases, characterised by $\bar{l}_p \sim 1$ and $e^{i2k\pi/N}$ repectively,

$$\sigma_W^{0,k} = k(N-k)\frac{4\pi^2T^2}{3\sqrt{3\lambda(T)}}(1 + O(g^2N)) \tag{3.18}$$

where $\lambda(T) \equiv g^2(T)N$ is the 't Hooft coupling that one keeps constant for a smooth large-N limit.

As we observed in [1] the deconfining phase transition appears to become more strongly first order as $N\uparrow$, and the finite-V corrections appear to become rapidly smaller, e.g. $h' \propto 1/N^2$ in eq. (3.12). Our more extensive calculations in this paper will confirm this. The tunnelling near the phase transition can be seen on smaller V as $N\uparrow$, and for a given V (in physical units) the tunnelling becomes rapidly rarer as $N\uparrow$. We interpret this as being due to the confining/deconfining interface tension, $\sigma_{c,d}$, growing with N. A reasonable guess is that for $T \simeq T_c$ the tunnelling between the $k=0$ and $k=N/2$ deconfined phases (say N is even), will effectively pass through a sheet of confined phase so that

$$\sigma_{c,d} \propto \frac{1}{2}\sigma_W^{k=0,k=N/2} \propto N^2 \tag{3.19}$$

using eq. (3.18). From eq. (3.17) we see that this means that the tunnelling has a probability

$$P_{c,d}(L,T) \propto e^{-\frac{cN^2L^2}{T}} \tag{3.20}$$

so that to see tunnelling equally easily we need to reduce the volume $L \propto 1/N$ as we increase N. This roughly describes what we shall see in our calculations.

The assumptions of our above argument can, in fact, be weakened considerably. All we need is that $\sigma_W^{k=0,k=N/2} \sim O(N^2)$ for $T \simeq T_c$, and then the fact that $\sigma_W^{k=0,k=N/2} \leq 2\sigma_{c,d}$ obtains for us eq. (3.19) as a lower bound on the N-dependence of the domain wall tension. We remark that the growth of domain wall surface tensions as a power of N appears to be a common phenomenon (see section 5) so it is no surprise that it also occurs in the case of the deconfining phase transition.

4. Results

4.1 Setting the scale

To set the scale of the lattice spacing in physical units we use the $T=0$ string tension, σ. We extract σ from the mass of the lightest flux loop that winds around the spatial torus, which can be obtained from correlations of smeared Polyakov loops as described, for example, in [5]. We generally use spatial tori of length $l \equiv aL \geq 3/\sqrt{\sigma}$, since earlier calculations [5] suggest that finite volume corrections are then accurately given by just the leading universal string correction, i.e.

$$am_p(L) = a^2\sigma L - \frac{\pi}{3L} \tag{4.1}$$

assuming the confining string is a simple bosonic string as suggested by [5, 13].

We have performed string tension calculations for all the SU(N) groups discussed in this paper. The calculations are taken from a study of improved gluonic lattice operators [14] and we use a type of blocking/smearing that is much more effective than in earlier calculations. This leads to significant changes in earlier estimates of the continuum value of $T_c/\sqrt{\sigma}$ for both SU(2) and SU(3).

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SU(2)			SU(3)		
β	L	$a\sqrt{\sigma}$	β	L	$a\sqrt{\sigma}$
1.8800	4	0.773(16)	5.6925	8	0.3970(19)
2.1768	8	0.5149(77)	5.6993	8	0.3933(16)
2.2986	10	0.3667(18)	5.7995	10	0.3143(14)
2.3726	12	0.2879(10)	5.8945	12	0.2607(11)
2.4265	16	0.2388(9)	6.0625	16	0.19466(73)
2.5115	20	0.17680(76)	6.3380	24	0.12946(61)

Table 1: SU(2) and SU(3) string tensions calculated at the indicated values of β from confining strings of length aL that wind around the spatial torus.

SU(4)			SU(6)			SU(8)		
β	L	$a\sqrt{\sigma}$	β	L	$a\sqrt{\sigma}$	β	L	$a\sqrt{\sigma}$
10.637	10	0.3254(11)	24.500	10	0.3420(19)	43.85	8	0.3646(25)
10.789	12	0.27015(86)	24.845	12	0.2801(13)	44.00	10	0.3406(20)
11.085	16	0.19871(85)	25.452	16	0.20992(82)	44.35	10	0.2991(20)
						44.85	12	0.2596(24)

Table 2: SU(4), SU(6) and SU(8) string tensions calculated at the indicated values of β from confining strings of length aL that wind around the spatial torus.

Our calculated values of $a\sqrt{\sigma}$ are listed in table 1 and table 2. Once we determine the critical coupling $\beta_c(\infty)$ for a given value of L_t, we interpolate our values of $a\sqrt{\sigma}$ to that value of β and this provides our lattice estimate of T_c

$$\frac{aT_c}{a\sqrt{\sigma}} = \frac{T_c}{\sqrt{\sigma}} = \frac{1}{L_t a(\beta_c)\sqrt{\sigma}}; \qquad a(\beta_c) = \frac{1}{L_t T_c}. \tag{4.2}$$

Repeating the calculation for various L_t, we can attempt to extrapolate the results to the continuum limit using a leading $O(a)$ lattice correction

$$\frac{T_c(a)}{\sqrt{\sigma(a)}} = \frac{T_c(a=0)}{\sqrt{\sigma(a=0)}} + ca^2\sigma \tag{4.3}$$

where c is some unknown quantity that is fitted.

4.2 Locating the phase transition

To locate a first order transition we search for values of β at which there is tunnelling between the confined and deconfined phases. (See e.g. figure 1 and figure 6 of [1].) This is easiest to do on smaller volumes where the tunnelling is sufficiently frequent that it can be seen in modest Monte Carlo runs. (Although if the volume is too small the transition will be completely washed out.) One can then use the $\beta_c(V)$ extracted from a smaller volume as a starting point for calculations at larger V.

We typically carry out $O(200K)$ sweeps at each β and have 2 to 5 useful values of β. They will be close enough together that the distributions of $\bar{u}_p$ have a strong overlap, and we then use standard reweighting techniques [15] to produce $\chi_l(\beta)$ and $C(\beta)$ as continuous

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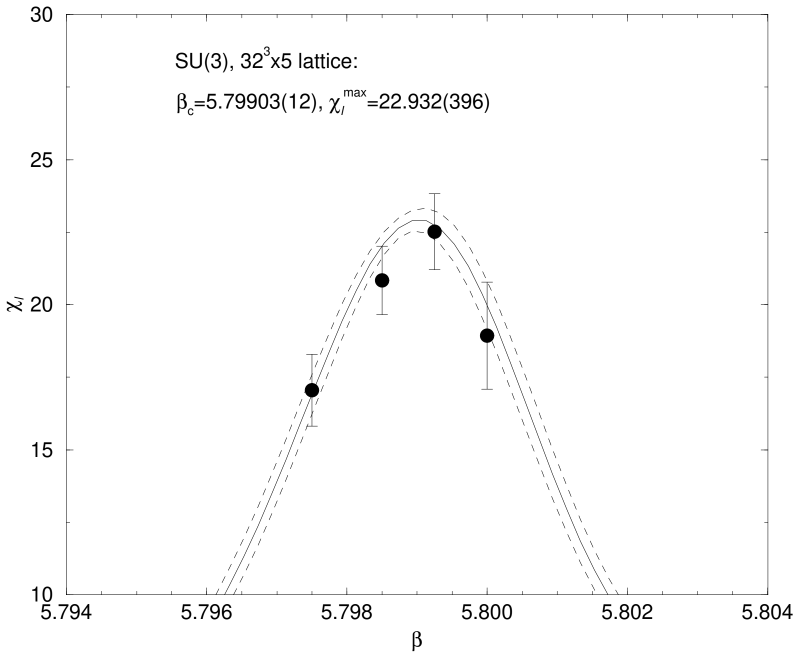

Figure 1: Reweighted susceptibility χ_l as a function of β for SU(3) on a $L = 32$ lattice at $L_t = 5$. The location of the maximum, β_c, and the value of the maximum, $\chi_l^{\max}$, are given as well.

functions of β and to obtain an accurate value for the position of the maximum, $\beta_c^C(V)$ or $\beta_c^\chi(V)$. An example is shown in figure 1. We then repeat the calculation for various volumes, L^3, at a fixed value of L_t and perform a finite size study (see below) to obtain $\beta_c(\infty)$, using eq. (3.13), the latent heat, using eq. (3.8), and so on. We then proceed to the lattice and continuum values of $T_c/\sqrt{\sigma}$ as described in section 4.1.

In practice we carry out the finite volume study at one value of L_t for each value of N and then use the parameter h obtained from a fit of the form in eq. (3.13) at the other values of L_t where we perform calculations typically on just one volume. This is the procedure used in past SU(3) calculations, e.g. [11], but is dangerous because the corrections to h in eq. (3.13) are not merely $O(a^2)$, as they would be for h' in eq. (3.12), but include the violations of asymptotic scaling in the $d\ln a/d\beta$ factor. This is a weakness of this calculation, which can only be remedied by a study on a much larger scale than ours.

For the case of SU(2), where the transition is second order, it turns out that the specific heat $C(\beta)$ has no visible peak as we vary β and its value does not grow with V. This anomalous behaviour arises because the plaquette has almost no overlap onto any correlation length that diverges. This interesting behaviour will be discussed in more detail in [2]. Here we merely note that one can instead focus upon the Polyakov loop susceptibility χ_l, as is usually done, and that exhibits all the characteristics of a second order phase transition.

4.3 Finite size study at $L_t = 5$

We perform our finite-V studies at $L_t = 5$ for all the values of N that we investigate. For a quick global view of what is going on we plot in figure 2 the value of $\ln\chi_l$ (extracted at

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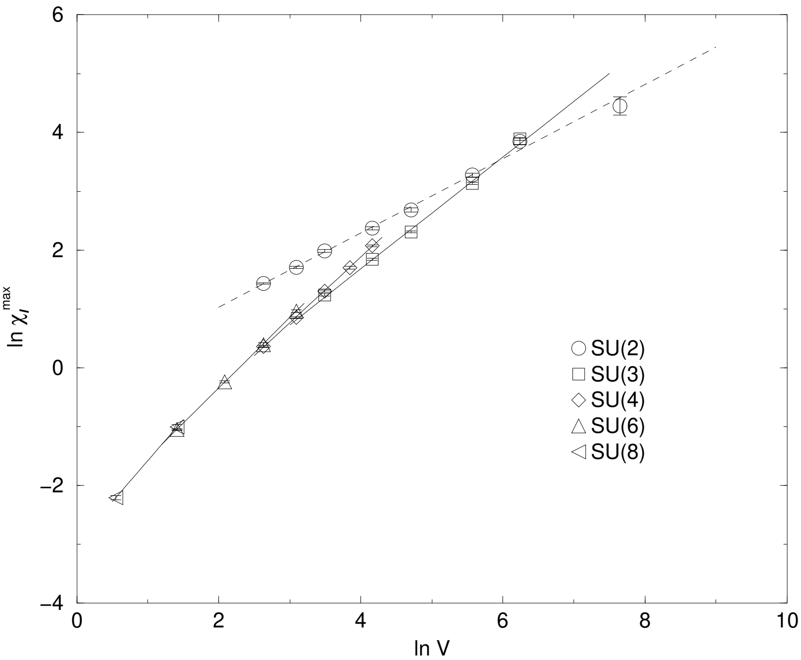

Figure 2: The value of $\ln \chi_l^{\max}$ plotted versus $\ln V$ for all our values of N at $L_t = 5$. The full lines are best fits for $N = 3, 4, 6, 8$ with a slope close to unity while the dashed line is the best fit for $N = 2$ with a slope of ~ 0.63.

its peak value) against $\ln V$ for all our values of N. We see that for $N \geq 3$ the values fall approximately on a straight line of slope unity. This is just the slope appropriate to a first order transition (see eqs. (3.9), (3.11)) and is in contrast to the SU(2) line that is flatter (with a slope of ~ 0.63) indicating that the transition there must be second order.

To see the corrections, a plot of the normalised susceptibility maxima $\chi_l^{\max}/V$ versus $1/V$ is more useful, and this we show in figure 3 for SU(2) and SU(3). We see that for SU(2) $\chi_l^{\max}/V \to 0$ as $V \to \infty$ indicating it is second order. (Our largest SU(2) lattice is $L = 64$.) For SU(3) the behaviour at smaller V is similar to that of SU(2), indicating a growing correlation length, but eventually it turns upwards, showing that it is indeed dominated by a first-order discontinuity. So SU(3) is 'weakly' first order. For $N \geq 4$ (figure 4) the picture is much less ambiguous: there is the expected correction that is linear in $1/V$ with a flattening only at small V where presumably the correlation length is being felt. As $N \uparrow$ the values of V where the flattening occurs is at ever smaller V indicating that the strength of the first order transition is growing (in the sense that the discontinuity dominates over the contribution of the longest correlation length at ever smaller V).

In figure 5 we perform similar plots for the normalised specific heat maxima $C_{\max}/V$. We see a much nicer linear behaviour that is easy to extrapolate. The reason for this is that, unlike χ_l, $C(\beta)$ is not sensitive to the lightest correlation length (which is proportional to $\sigma(T)$). Indeed we do not show the SU(2) curve, because we cannot identify a maximum — and if we did its value would be independent of V (in our range of V).

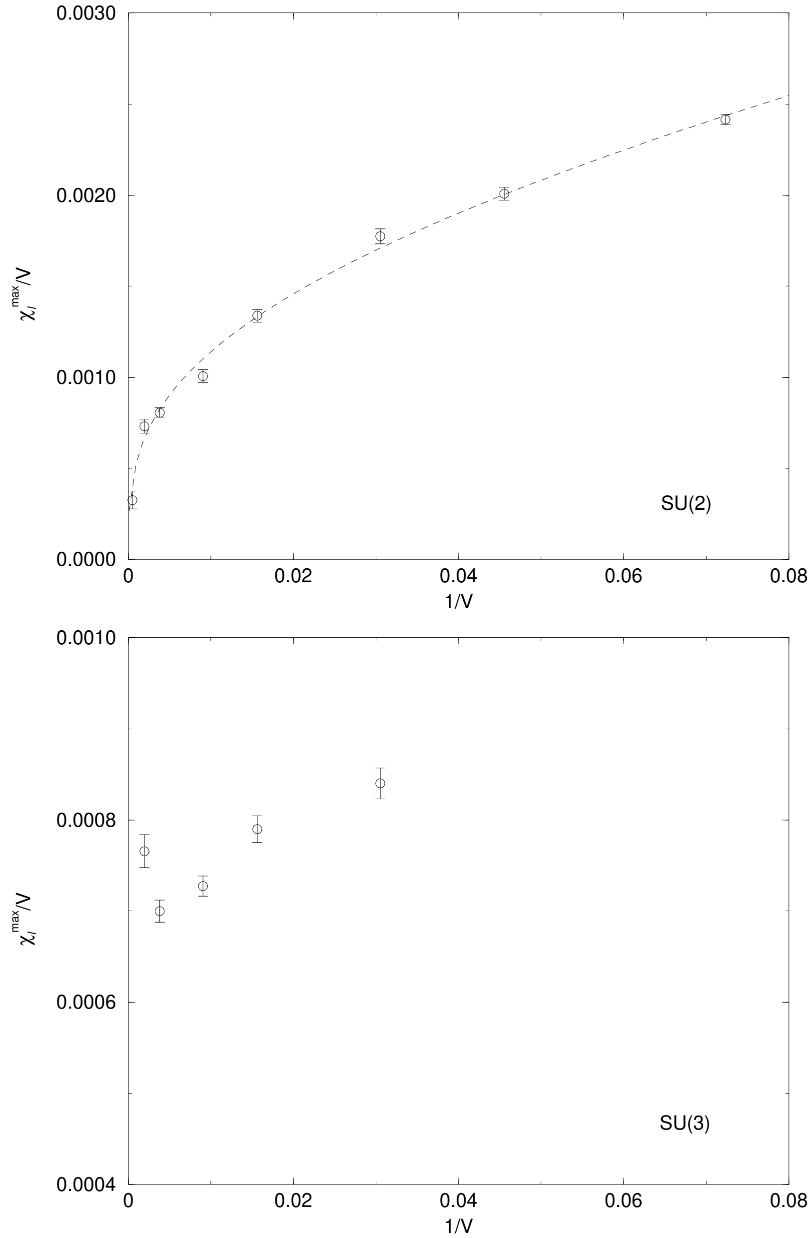

Figure 3: The normalised susceptibility maxima $\chi_l^{\max}/V$ plotted against $1/V$ for $N = 2, 3$ at $L_t = 5$. The dashed line is a best fit with the leading scaling behaviour extracted from figure 2 and a leading $O(1/V)$ correction.

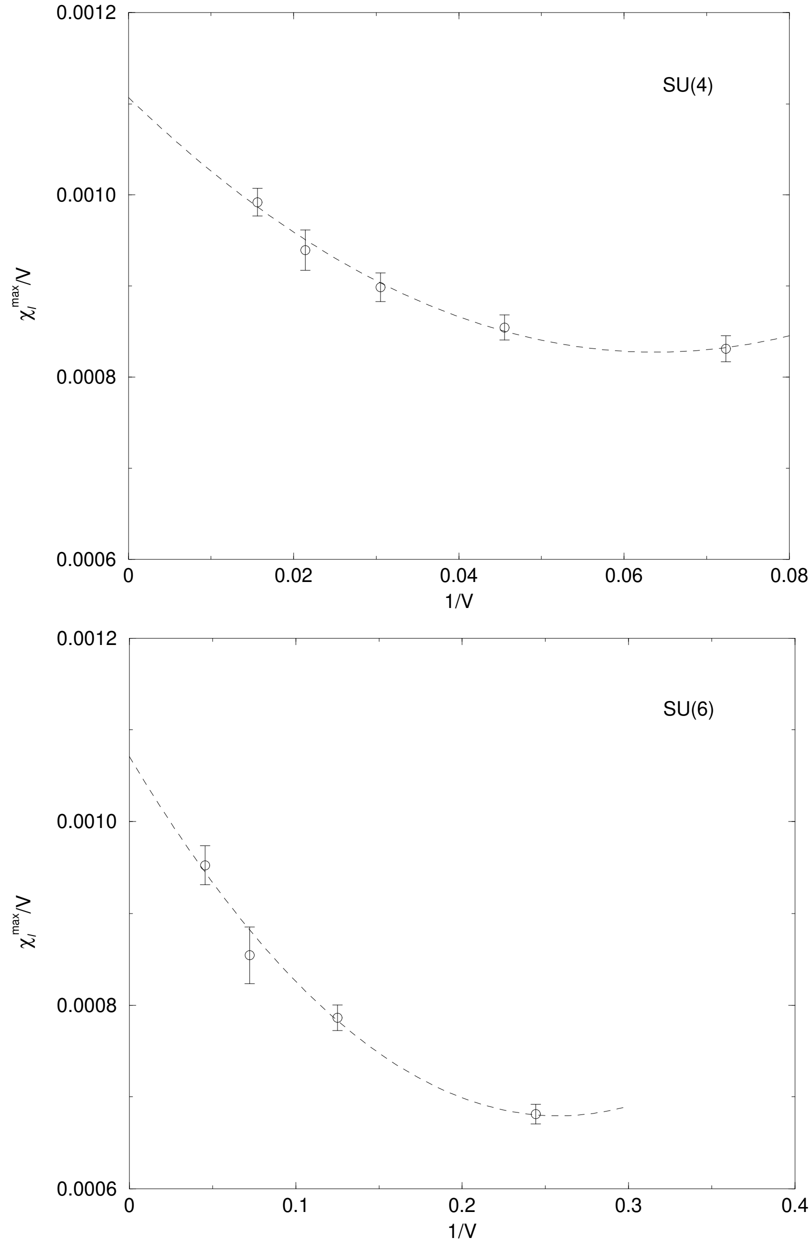

Figure 4: The normalised susceptibility maxima $\chi_l^{\max}/V$ plotted against $1/V$ for $N = 4, 6$ at $L_t = 5$. The dashed lines are the best fits with $O(1/V)$ and $O(1/V^2)$ corrections. Note the different scale for SU(6) on the x-axis.

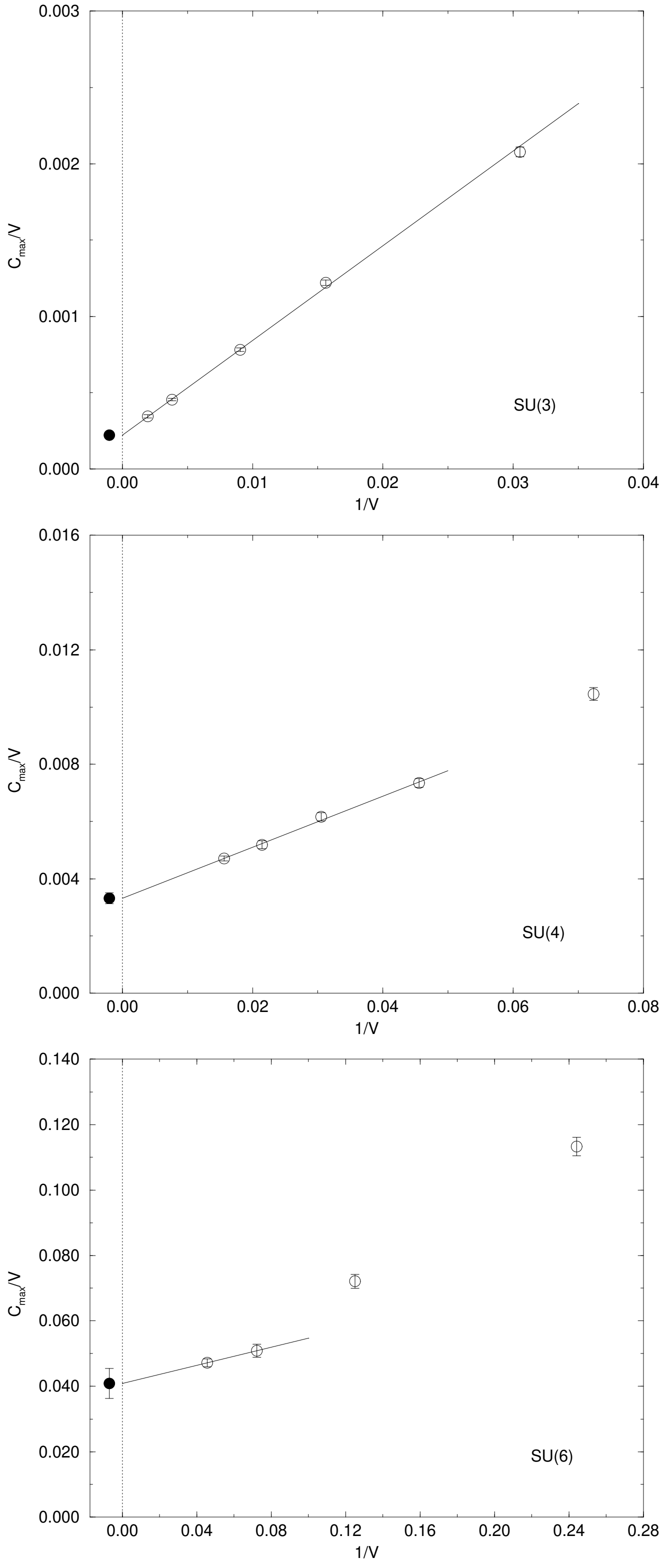

Figure 5: The normalised specific heat maxima $C_{\max}/V$ plotted against $1/V$ for $N = 3, 4, 6$ at $L_t = 5$. The straight lines are best fits with a leading $O(1/V)$ correction. Also shown are the $V \to \infty$ limits.

We can use the maximum of χ_l to define $\beta_c(V)$ and from a fit to eq. (3.13) we can extract h, the coefficient of the $1/V$ correction. This is tabulated in table 3. We recall that the usually quoted SU(3) value is $h \leq 0.1$ [11]. We find it impossible to extract a reliable value of h from our values of $\beta_c(V)$ at $L_t = 5$ (although we have some calculations for $L_t = 4$ that agree precisely with earlier calculations) and we put that down to the weakness of the SU(3) transition — or perhaps a large statistical fluctuation in our calculations. In any case, it is already clear from table 3 that we do not have $h \propto N^2$ as one would need for a finite h' in the $N = \infty$ limit. (See eqs. (3.12), (3.13), (3.14).) Rather it looks as though $h' \propto 1/N^2$.

	h	$L \in$	χ^2/n_{df}
SU(4)	0.090(17)	[12,20]	0.49
SU(6)	0.112(19)	[8,14]	0.04
SU(8)	0.063(70)	[6,8]	–

Table 3: The coefficient h of the $1/V$ correction to $\beta_c^{\chi}(V)$ for the groups shown, obtained from fits (best χ^2 shown) to the range of volumes shown.

4.4 Latent heat

We see from eq. (3.8) that we can obtain L_h, the latent heat per plaquette, from the $V = \infty$ extrapolation of $C_{\max}/V$. Doing so for our $L_t = 5$ calculations, we obtain the value of L_h for each N and we plot the results in figure 6.

The first thing we observe is that L_h grows with N. Moreover the growth is consistent with being due to a $O(1/N^2)$ correction, as shown by the dashed line in the plot. It is clearly no surprise that the latent heat vanishes for $2 < N < 3$, as exemplified by our fit, so that the $N = 2$ transition is second order.

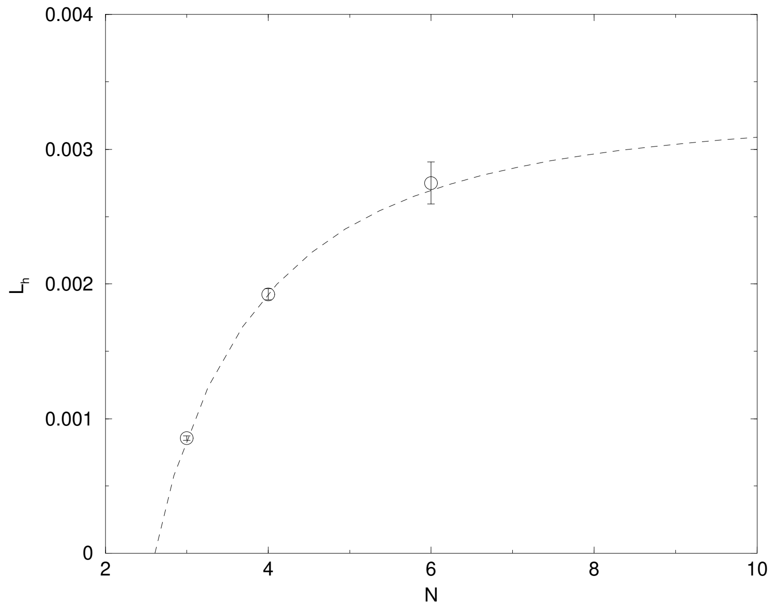

Figure 6: The latent heat L_h, as defined in eq. (3.8), plotted versus N at $L_t = 5$. The dashed line is a large-N extrapolation with a leading $O(1/N^2)$ correction. Because of the additional factor $\beta = 2N/g^2 = 2N^2/\lambda$ in the action (λ being the 't Hooft coupling) the physical latent heat is $\propto N^2 L_h$.

SU(2)			SU(3)		
L_t	β_c	$T_c/\sqrt{\sigma}$	L_t	β_c	$T_c/\sqrt{\sigma}$
2	1.8800(30)	0.647(15)	–	–	–
3	2.1768(30)	0.6474(111)	4	5.69236(15)	0.6296(30)
4	2.2986(6)	0.6818(35)	5	5.8000(5)*	0.6370(29)
5	2.37136(54)*	0.6918(29)	6	5.8941(12)	0.6388(32)
6	2.4271(17)*	0.6994(50)	8	6.0625(18)	0.6421(31)
8	2.5090(6)*	0.7008(34)	12	6.3385(55)	0.6442(51)

Table 4: Critical values of β, extrapolated to $V = \infty$, for SU(2) and SU(3) for the values of L_t shown; with the corresponding values of the deconfining temperature in units of the string tension. Starred values are new calculations of β_c.

	SU(4)		SU(6)		SU(8)	
L_t	β_c	$T_c/\sqrt{\sigma}$	β_c	$T_c/\sqrt{\sigma}$	β_c	$T_c/\sqrt{\sigma}$
5	10.63709(72)	0.6146(23)	24.5139(24)	0.5894(36)	43.978(22)	0.5814(68)
6	10.7893(23)	0.6171(26)	24.8458(33)	0.5952(32)	44.496(3)	0.5807(71)
8	11.0848(23)	0.6289(31)	25.4712(62)	0.6009(29)	45.606(35)	0.5964(154)

Table 5: Critical values of β, extrapolated to $V = \infty$, for SU(4), SU(6) and SU(8) for the values of L_t shown; with the corresponding values of the deconfining temperature in units of the string tension.

The lattice latent heat is $O(a^4)$. Can a variation of a with N account for the variation we see in figure 6? If we express a in units of T_c then the answer is of course no: by definition $a = 1/L_tT_c = 1/5T_c$ is fixed for all N. If instead we use σ to set the scale, then a^4 does indeed vary as we go from $N = 3$ to $N = 6$, but only by about $\sim 30\%$ which is a very small part of the observed change. Thus we conclude that in physical units the latent heat increases strongly as we go from $N = 3$ to larger N, indicating that the first order transition strengthens as $N \uparrow$.

Finally we remark that because of the additional factor of $\beta = 2N/g^2 = 2N^2/\lambda$ in the action, where $\lambda \equiv g^2N$ is the 't Hooft coupling kept constant as $N \to \infty$, the physical latent heat is $\propto N^2L_h$. (L_h as defined in eq. (3.8) is really the latent heat per degree of freedom.) Since we find that L_h goes to a non-zero constant as $N \to \infty$, this means that the physical latent heat is $O(N^2)$ at large N (so that it is proportional to the gluon condensate [16]).

4.5 $T_c/\sqrt{\sigma}$ for all N

We list our values of $\beta_c(V = \infty)$ in table 4 and table 5. The SU(2) values include older calculations [17] as do the SU(3) ones [11, 12]. The (interpolated) string tensions are new and more accurate than those used previously. For our SU(4) and SU(6) calculations at $L_t = 6, 8$ we use the h extracted at $L_t = 5$. However we double the error so as to (hopefully) cover any change of h with L_t. For SU(8) we also extract h for $L_t = 5$, but because the error on it is already very large, we feel no need to increase it at larger L_t.

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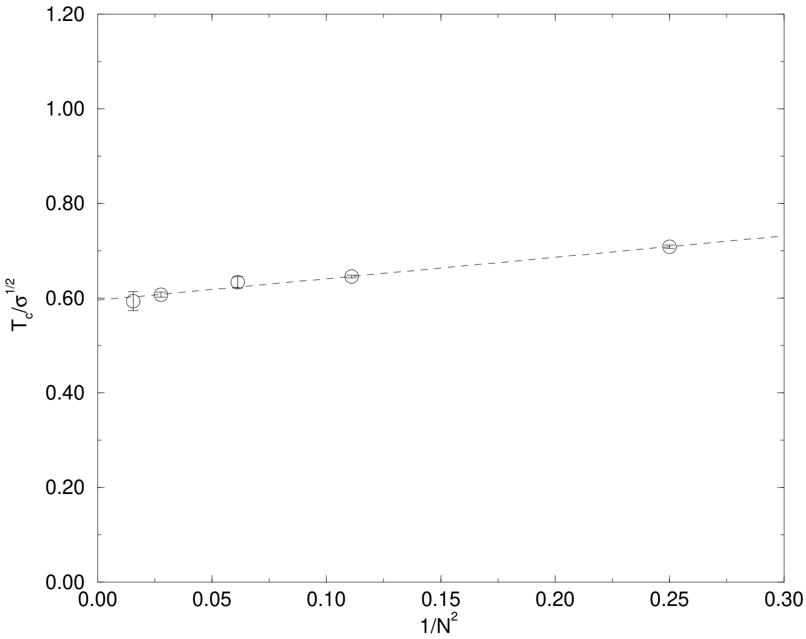

Figure 7: The deconfining temperature, T_c, in units of the string tension, σ plotted versus $1/N^2$. The dashed line is the large-N extrapolation with a leading $O(1/N^2)$ correction.

We use eq. (4.2) to obtain the values of $T_c/\sqrt{\sigma}$ shown, and then use eq. (4.3) to obtain the continuum extrapolation, as tabulated in table 6. (In the SU(4) case where the best χ^2 is very poor, we use an error that covers the extrapolated values obtained using any two β values out of the three.)

Finally we take our various continuum values of $T_c/\sqrt{\sigma}$ and plot them against $1/N^2$ in figure 7. We see that the values are all well described by a leading large-N correction of the expected [3] functional form:

	$\lim_{a\to 0} T_c/\sqrt{\sigma}$	$L_t \in$	χ^2/n_{df}
SU(2)	0.7091(36)	[3,8]	0.28
SU(3)	0.6462(30)	[4,12]	0.05
SU(4)	0.634(12)	[5,8]	2.05
SU(6)	0.6078(52)	[5,8]	0.00
SU(8)	0.594(20)	[5,8]	0.53
SU(∞)	0.5960(41)	–	0.29

Table 6: Continuum extrapolations of $T_c/\sqrt{\sigma}$ with the range of L_t values used and the χ^2 per degree of freedom of the fit. Also shown is the result of the $N \to \infty$ extrapolation of these values, as described in the text.

$$\frac{T_c}{\sqrt{\sigma}} = 0.596(4) + \frac{0.453(30)}{N^2}\,. \tag{4.4}$$

The coefficient is modest, indicating that for this physics at least, all values of N are close to $N = \infty$, despite the fact that SU(2) is second order while larger groups are first order. Only the latent heat appears to have large corrections in N.

4.6 V-dependence as $N \uparrow$

The values of h in table 3 strengthen our earlier observation [1] that h is roughly constant indicating that the finite volume corrections disappear as $\propto 1/N^2$.

It is also clear from e.g. figures 3 and 4 that the first order character of the transition becomes clearer on ever smaller volumes as $V \uparrow$. We have interpreted this as being related

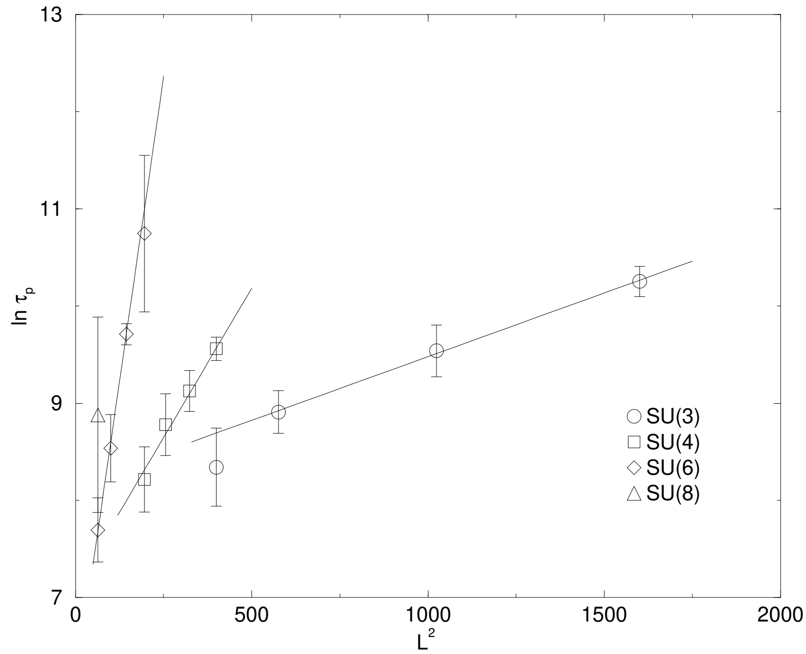

Figure 8: The persistence time, τ_p, plotted versus L^2 for various N at $L_t = 5$. The straight lines are fits of the form $\tau_p \propto \exp -c(N)L^2$.

to the domain wall surface tension growing with N, perhaps as in eqs. (3.18), (3.19). To analyse this further we have roughly calculated the number of sweeps that the lattice spends in each of the confined and deconfined phases when it is very close to $\beta_c(V)$. We call this number the persistence time, τ_p. From the statistical errors on our $T = 0$ string tension calculations, we infer that the efficiency of our Monte Carlo in decorrelating physical fluctuations does not vary much with N, so that comparing τ_p across N is physically meaningful. We plot τ_p for various volumes and various N in figure 8, against L^2. We see that it is consistent with being $\tau_p \propto \exp -(N^\alpha L)^2$ with α unity or greater. In fact the L dependence is not really well determined: a plot against L^3 looks equally good. If we simply take the largest L where we have roughly equal τ_p for $N = 3, 4, 6$ i.e. $L = 32, 20, 12$ respectively, then a trend to be approaching a function of $N \times L$ for larger N is certainly consistent. However this analysis has to be regarded as very preliminary at this stage, and only indicative of what one might eventually achieve [2].

5. Some conjectures about $N = \infty$

5.1 Reduced models

On a given spatial volume, $V = L^3$, at fixed L_t and at values of β close to the corresponding phase transition, we have found the tunnelling to be rapidly suppressed as $N \uparrow$. Roughly speaking if one wants to keep the tunnelling rate constant one needs to decrease the volume as $L \propto 1/N^\alpha$, where α appears to be at least unity. This is roughly consistent with our conjecture that the surface tension of the confining/deconfining domain wall varies as

$\sigma_{c,d} \propto N^2$, and in any case that this provides a lower bound on its variation with N. We also saw that the shift of $aT_c(V)$ from $aT_c(\infty)$ appears to be $O(1/N^2)$, suggesting that as $N \to \infty$ one can locate the transition on volumes $V \to 0$.

As we pointed out in [1] this cannot really be the case, because once $L < 1/a(\beta)T_c \simeq L_t$ the spatial version of the center symmetry in eq. (3.6) will become spontaneously broken. For example if we are considering $L_x < L_t$ then the relevant symmetry is as in eq. (3.6) but with U_x replacing U_0, n_x replacing n_t, and n_s counting the points in the (y, z, t) 3-slice. This breaking occurs for the same reason that the symmetry is broken in the temporal direction for $L_t < 1/a(\beta)T_c$; after all the label of the euclidean direction does not matter. (For finite N this breaking will itself be smoothened by finite volume effects; however as $N \to \infty$ the observed loss of finite-V corrections suggests it will become precise, even on a finite volume.) For this reason we conjectured in [1] that as $N \to \infty$ the deconfining transition will occur at exactly the same β_c for all volumes $L > L_t = 1/a(\beta)T_c$, i.e. the deconfining physics is independent of the volume down to a critical volume, and we speculated upon the relation of this to Witten's idea of an $N = \infty$ Master Field that is necessarily translation invariant [18]. We also note the relation of this to recent conjectures [19] about how the Eguchi-Kawai reduction [20] translates to the continuum.

Of course, as $N \to \infty$ the transitions on this minimal V will presumably become rapidly rarer and the transition will become ever harder to locate. Here we speculate on a way of reducing V below this minimal value. This is based on the observation that if we impose an $\{x, t\}$ twist for $T > T_c$, it induces a corresponding Z_N domain wall separating two of the Z_N deconfined phases. That is to say, it disorders the Z_N symmetry breaking. Indeed this domain wall can be interpreted [21] as a 't Hooft dual vortex in its 'Higgs' phase. If L_x becomes small there is no room for the two phases and the wall, and we expect that the corresponding symmetry breaking is suppressed. Thus we conjecture that on an $L^3 L_t$ volume at β_c, if we reduce $L < L_t$ but introduce a (maximal) twist in $\{x, y\}$, $\{x, z\}$ and $\{y, z\}$ we suppress the symmetry breaking and can continue to reduce L, perhaps as $1/N$, while maintaining the physics of the deconfining transition. As $N \to \infty$ we can therefore study T_c on an infinitesimal volume: in a sense on a single spatial point. And perhaps other physical quantities can also be so studied. This reduction connects naturally to the twisted Eguchi Kawai reduced model [20, 22].

5.2 Master field(s)

The factorisation of gauge invariant operators at large-N, and the fact that fluctuations vanish in leading order, led to an old speculation [18] that in the $N = \infty$ limit there is only a single Master field in the path integral (which is necessarily translation invariant for gauge invariant quantities). It has long been realised that this picture is deceptively simple (e.g. [23]).

In the case of interest here, we have seen that the fluctuations near $T = T_c$ between the confined and deconfined phases, are suppressed exponentially in N^2 (at least) on a volume that is fixed in physical units as $N \uparrow$. If one wishes to talk of a 'Master Field' one has to talk of at least 2: one confined and one deconfined. (For the $D = 3+1$ euclidean system there are presumably $N \to \infty$ deconfined Master Fields, separated by infinite barriers in that limit.)

We note that as T deviates from T_c the difference in free energy density between the confined and deconfined vacua will be $O(N^2)$ and for large enough bubbles will outweigh the suppression of the domain walls (if that is also $O(N^2)$ as we have suggested). However the system has to pass through small bubbles to reach these large bubbles, and the probability of these will continue to be dominated by the exponential suppression of the surface area of the bubble. That is to say, one can imagine a scenario at $N = \infty$ in which there is no transition between confining and deconfining phases at any value of T — a kind of infinite super-cooling/heating.

This picture of different phases separated by infinite barriers at $N = \infty$ is also characteristic of the lattice bulk transition [2, 6] and of the multiple vacua inferred from the θ-dependence of the SU(N) gauge theory [24, 25]. All this suggests that the gauge theory possesses an infinite number of 'Master Fields' that mutually decouple precisely at $N = \infty$ but whose simultaneous existence ensures a rich physics of that limiting theory.

6. Conclusions

In this paper we have obtained some properties of the deconfining transition in SU(N) gauge theories. We have done so for enough values of N that we feel confident in our control of the $N \to \infty$ limit. At the same time, our dedicated string tension calculations allow us to improve significantly upon older continuum extrapolations for SU(2) and SU(3).

We find the phase transition is first order for $N \geq 3$, and that the latent heat (of the $D = 3 + 1$ euclidean system) grows with N, as shown in figure 6, showing that the transition becomes more strongly first order as $N \uparrow$. Indeed it is clear from figure 6 that the SU(3) transition is quite weakly first-order, and that a naive interpolation of the latent heat as a function of N predicts that the transition ceases to be first order somewhere between $N = 2$ and $N = 3$.

We also find that as N grows the phase transition becomes clearly defined on ever smaller volumes (see e.g. figures 3–5). We argue that this is due to the interface tension between the confined and deconfined phases growing at least as rapidly as N^2. At the same time we find that the finite-V corrections to T_c appear to be $O(1/N^2)$ so that at $N = \infty$ T_c can be calculated on small volumes. We speculated in section 5 how one might be able to take the limit $V \to 0$ in a way that is reminiscent of the twisted Eguchi-Kawai model. We also speculated on how this might fit in with the old idea that $N = \infty$ is dominated by a Master Field that is translation invariant for gauge invariant observables. We plan to test the idea in section 5.1, not only for T_c but for other physical quantities as well.

The values of $T_c/\sqrt{\sigma}$ that we obtain in the continuum limit show a modest N-dependence, displayed in figure 7. This dependence can be well fitted with a leading $O(1/N^2)$ correction all the way down to SU(2), as in eq. (4.4). Such modest corrections are in fact characteristic of the N dependence of many physical quantities [6, 26].

We remark that there exist old calculations of the large-N deconfinement transition within the twisted Eguchi-Kawai framework (e.g. [27] and references therein) which found the transition to be first order (to the extent that this can be established without a proper finite size scaling study). While a serious attempt at obtaining the continuum limit of

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$T_c/\sqrt{\sigma}$ was not possible given the computing resources then available, some of the values obtained of $T_c/\sqrt{\sigma}$ at finite a were consistent with our calculations, albeit within very large statistical errors. The large discrepancy between the values quoted by different groups was largely due to a lack of control over the calculation of the string tension σ. (See [28] for a review.) It would clearly be interesting to return to such calculations using modern techniques and computer resources.

As we have stressed here and in [1] there are many ways in which our calculations need to be improved. Perhaps their main weakness lies in the control of finite-V corrections. Because such studies are expensive we have followed the usual strategy of performing a finite-V study only at the coarsest value of the lattice spacing a and then using that to estimate finite-V corrections at other values of a. That this is likely to be dangerous is indicated by the nature of the theoretical expression, eq. (3.14). Another weakness concerns our latent heat calculation which, being coupled to the finite-V study, has again only been performed for the coarsest value of a. Here in fact we can do better, using a different technique, and in [2] we will do so. In that paper we will also present calculations of the correlation lengths as $T \to T_c$, obtained separately for the confined and deconfined phases. This will provide another measure of the strength of the transition. In addition we will present some results for the physical latent heat of the gauge theory (which is different from the 'euclidean' latent heat that we discuss in this paper), for the way topological fluctuations change across $T = T_c$, and we will present the results discussed in this paper in greater detail.

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